ARTICLE NO. 78-893R

Periodic Heat Transfer in Radiating and Convecting Fins or Fin Arrays

Ralph G. Eslinger* and Benjamin T. F. Chung†
University of Akron, Akron, Ohio

A finite-element solution is presented for the heat transfer from radiating and convecting fins or fin arrays. Consideration is given to thin, straight fins attached to a base surface for which the temperature varies periodically. Fin-to-fin, fin-to-base, and fin-to-environment radiative interactions are considered. It is assumed that the radiating surfaces are gray, the environment is black, and the surrounding fluid is transparent. An absorption factor technique is employed for the computation of energy exchange within the nonisothermal enclosure of the fin array.

Nomenclature

A = dimensionless amplitude parameter
A_f = surface area fin
B = dimensionless frequency parameter, $\omega L^2 k/\rho c_p$
B_{ne} = absorption factor
b = fin thickness
$[C]_e$ = elemental capacitance matrix
$[C]$ = global thermal capacitance matrix
c_p = specific heat
$[D]_e$ = displacement matrix,

$$[D]_e = \begin{bmatrix} 0 & 0 \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & \cdot \\ 0 & 1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ \\ i\text{th row} \\ j\text{th row} \\ \\ \\ \\ \\ \end{matrix}$$

d = base length between adjacent fins
$dB(x'x)$ = differential absorption factor
F = functional to be minimized
F_e = elemental form of functional to be minimized
$\{g\}_e$ = elemental internal heat generation column vector
$\{g\}$ = internal heat generation column vector
$[H]_e$ = elemental global convection matrix
$\{h\}$ = convection column vector
I = variational statement
I_e = elemental form of variational statement
$[K]_e$ = elemental global conduction matrix
$[K]$ = global conduction matrix
k = thermal conductivity
L = length of fin
L' = length of the fin immediately adjacent to the fin under consideration
m = number of fin elements
N_c = convection number, hL^2/bk
N_r = radiation number, $\epsilon\sigma t_b^3 L^2/2bk$
p = numerical step size parameter
$[P]^T$ = $[1 \quad x]$
$[P]_e$ = $\begin{bmatrix} 1 & x_i \\ 1 & x_j \end{bmatrix}$
Q = total heat transfer from the fin
$[R]_e$ = elemental radiation matrix
$[R]$ = global radiation matrix
$\{r\}_e$ = elemental radiation column vector
$\{r\}$ = radiation column vector
t = temperature
$\{t\}_e$ = elemental temperature vector
$\{t\}$ = column vector of nodal temperatures
$\{\dot{t}\}$ = column vector of temperature time derivatives
t_a = an environment temperature
t_i = temperature at location x_i on the fin
t_j = temperature at location x_j on the fin
t_m = mean temperature of base surface
t_x = first derivative of temperature with respect to location
t_θ = first derivative of temperature with respect to time
u''' = internal heat generation
x = space coordinates of fin
x_{ij} = designates $x_j - x_i$
σ = Stefan-Boltzmann constant
ϵ = surface emissivity
θ = time
δ_{en} = Kronecker delta
ρ = fin density
ω = circular frequency of base temperature oscillation
α = thermal diffusivity
λ = convergence interval
ν = time step designation
η_r = radiation fin efficiency
η = combined mode fin efficiency
$[\ ]$ = $\sum_{e=1}^{m} [D]_e[\ ]_e[D]_e^T$, global matrix
$[\ ]_e$ = elemental matrix
$\{\ \}$ = $\sum_{e=1}^{m} [D]_e\{\ \}_e$, column vector
$\{\ \}_e$ = elemental column vector
$[\ ]^T$ = transpose of a matrix
$[\ ]^{-1}$ = inverse of a matrix

Received Feb. 2, 1978; presented as Paper 78-893 at the 2nd AIAA/ASME Thermophysics and Heat Transfer Conference, Palo Alto, Calif., May 24-26, 1978; revision received April 2, 1979.
Index categories: Heat Conduction; Radiation and Radiative Heat Transfer; Analytical and Numerical Methods.
*Presently, Supervisory Engineer, Bendix Corp. Member AIAA.
†Associate Professor, Mechanical Engineering Department.

Introduction

In many practical engineering applications, situations arise in which the heat transfer from a fin occurs neither in a steady nor transient manner, but rather as steady periodic heat transfer. Examples of the latter include cooling of electronic equipment, processes requiring that temperatures be maintained between specified limits, and cutting tools subjected to periodic operation.

The only steady periodic heat-transfer problems appearing in the literature, for which exact analytical solutions have been obtained, are given by Yang[1] and Aziz.[2] In each case, convection was assumed to be the only mode by which heat was dissipated from the fin.

Problems of heat transfer by radiation alone from a finned surface at steady-state conditions have been widely studied. In most of the investigations, a finite-difference formulation was employed and the radiosity method[3] was used to solve the problem of multiple reflections within a gray, nonisothermal enclosure. Schnurr and Cothran,[4] and Sparrow and coauthors[5-7] have investigated radiating fin problems of a steady-state nature successfully.

This paper is concerned with heat transfer occurring by the combined modes of radiation and convection within an enclosure formed by adjacent fins, base surface, and the environment. Due to the nonlinear nature of the problem, the exact solution is not feasible. Hence, the finite-element method is employed for the solution. Obviously, the finite-difference method can be used. However, the finite-element method is more flexible in treating irregular shapes. The present computer program can be easily extended to deal with different geometries of the fin.

Since the first application of the finite-element method to thermal problems by Wilson and Nickell,[8] and many others[9-14] have successfully applied the technique to various boundary value problems of heat conduction. To the authors' knowledge, no previous work is available for which a finite-element analysis includes radiation in the energy equation, although the radiative boundary condition has been treated in an excellent text by Myers.[9] In the present analysis, Gebhart's[5] absorption factor method was selected to solve for the radiant interchange since it provides an expression explicitly in terms of the element temperatures.

Numerical solutions based on the finite-element analysis are presented in graphical form, with comparisons made to the exact analytic solutions for steady periodic heat transfer by convection alone and steady, nonperiodic heat transfer from a single fin radiating to free space.

Problem Formulation

Consideration is given to a thin fin with its base temperature varying periodically, as illustrated in Fig. 1. The primes in Fig. 1 are used to denote a fin immediately adjacent to the fin under consideration, and the symmetry of the system has been used to reduce the number of surfaces that have radiant interactions and need to be considered in the formulation. It is assumed that the fin or fin arrays are of semi-infinite extent. In addition, the following assumptions apply:

1) The base surface acts entirely as an isothermal surface.

2) All surfaces, with the exception of the environment which is assumed to be black, act as gray diffuse emitters and absorbers.

3) Radiation incident on any of the surfaces is uniformly distributed over the surface.

The energy equation applicable to the system shown in Fig. 1, using the concept of absorption factors, is given as:

$$\frac{\partial^2 t}{\partial x^2} - \frac{2h}{bk}[t-t_a] + \frac{u'''}{k} - \frac{\sigma}{bk}\left[\epsilon t^4 - \int_0^{L'} \frac{\epsilon' \mathrm{d}B(x',x)\, t^4 \mathrm{d}x'}{\mathrm{d}x} - \int_0^{L} \frac{\epsilon \mathrm{d}B(x,x)\, t^4}{\mathrm{d}x}\mathrm{d}x - \epsilon_b B(b,x)\, t_b^4 - \epsilon_a B(a,x)\, t_a^4\right] = \frac{1}{\alpha}\frac{\partial t}{\partial \theta} \qquad (1)$$

The associated boundary conditions are:

$$t(0,\theta) = t_b = t_m + (t_m - t_a)A\cos\omega\theta \qquad (2)$$

and

$$\frac{\partial t}{\partial x}(L,\theta) = 0 \qquad (3)$$

Where $B(x',x)$ is the absorption factor and is defined as the total fraction of the emission by x' which is absorbed by x. The radiation, which is absorbed at x and originates at x', consists of both direct irradiation from x' and radiation from x' which is absorbed at x, after having undergone multiple reflections within the enclosure. The absorption factors are functionally dependent on the configuration factors for the enclosure and exhibit similar reciprocity. The bracketed term in Eq. (1), which is multiplied by σ/bk, represents the radiant energy balance for a differential element of fin with the integral over L and L' accounting for the fin-to-fin interaction. At this point in the formulation, it will be convenient to recognize that the finite-element solution will be numerical in nature. The finite-element representation of the system is illustrated in Fig. 1. Proceeding to the finite-element model at this point does not require that $\mathrm{d}B(x',x)$ or $\mathrm{d}B(x,x)$ be given in exact differential form, but can be replaced with finite approximations. This is essential since the differential absorption factors do not have an explicit differential form, but are obtained as a finite surface that is subdivided into small finite sections. Introducing the notation that B_{ne} represents

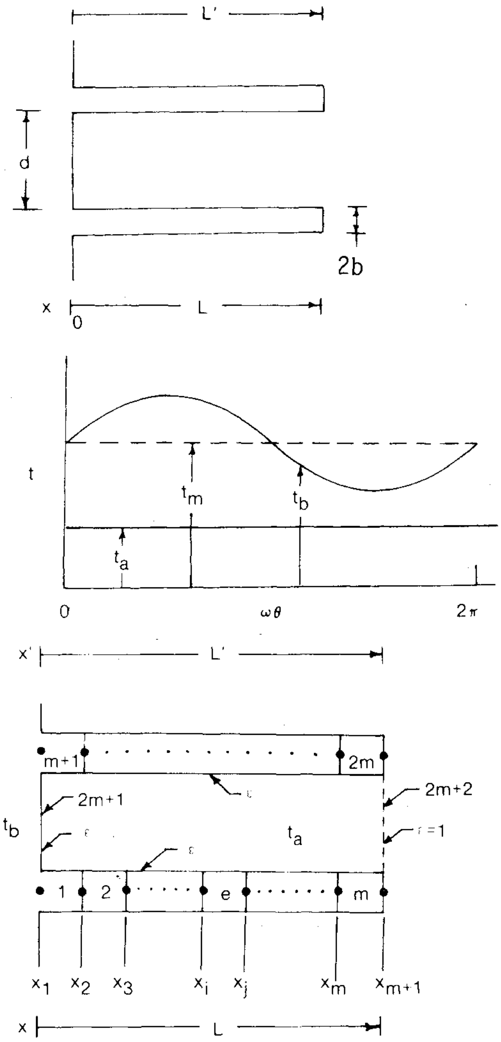

Fig. 1 Illustration of fin geometry, base temperature, and the finite-element representation of the enclosure.

the absorption factor from the nth surface of the $2m+2$ elements comprising the enclosure to the eth arbitrary element on the fin will allow for Eq. (1) to be rewritten as:

$$\frac{\partial^2 t}{\partial x^2} - \frac{h}{bk}[t-t_a] + \frac{u'''}{k} - \frac{\sigma}{bk}\left[\epsilon(1-B_{ee})t^4 - \sum_{n=1}^{2m+2}\delta_{en}\epsilon B_{en}t_n^4\right] = \frac{1}{\alpha}\frac{\partial t}{\partial \theta} \quad (4)$$

where the reciprocity relation for absorption factors given by

$$\epsilon' \mathrm{d}B(x',x)\mathrm{d}x' = \epsilon \mathrm{d}B(x,x')\mathrm{d}x \quad (5)$$

is used to allow for the cancellation of $\mathrm{d}x$ and the Kronecker delta is used to allow the unknown element temperature to be removed from within the summation. Equation (4) is representative of an elemental energy equation in which $\mathrm{d}B$ has been replaced with B_{en} to denote the arbitrary eth element of which the coordinates are given by x_i and x_j, as shown in Fig. 1. It is not necessary that the emissivity remain constant over the length of the fin. This will be subsequently shown as an elemental property in Eq. (16). From Eq. (4), it now becomes possible to determine an elemental expression for the functional F_e, the integral of which when minimized for each element and point in time will yield the temperature distribution along the fin.

The solution of Eq. (4) is equivalent to finding a function $t(x,\theta)$ which will minimize an integral of the form

$$I = \int_0^L F(x,\theta,t,t_x,t_\theta)\,\mathrm{d}x \quad (6)$$

at each location x and for every point in time. Equation (6) can be written in elemental form as:

$$I = \sum_{e=1}^{2m+2} I_e \quad (7)$$

where

$$I_e = \int_{x_i}^{x_j} F_e(x,\theta,t,t_x,t_\theta)\,\mathrm{d}x \quad (8)$$

From the calculus of variations, F_e in Eq. (8) is obtained as:

$$F_e = \frac{1}{2}kb\left[\frac{\partial t}{\partial x}\right]^2 + \frac{1}{2}h[t^2 - 2t_a t] + \frac{1}{5}\epsilon\sigma(1-B_{ee})t^5 - \sum_{n=1}^{2m+2}\delta_{en}\epsilon\sigma B_{en}t_n^4 t - u'''\,bt + \frac{1}{2}\rho c_p b\frac{\partial t^2}{\partial \theta} \quad (9)$$

Equation (9) can now be substituted into Eq. (8), giving the variational statement I_e as:

$$I_e = \int_{x_i}^{x_j}\left\{\frac{1}{2}kb\left[\frac{\partial t}{\partial x}\right]^2 + \frac{1}{2}h[t^2-2t_a t] + \frac{1}{5}\epsilon\sigma(1-B_{ee})t^5 - \sum_{n=1}^{2m+2}\delta_{en}\epsilon\sigma B_{en}t_n^4 t - u'''\,bt + \frac{1}{2}\rho c_p b\frac{\partial t^2}{\partial\theta}\right\}\mathrm{d}x \quad (10)$$

Having obtained the elemental expression for the variational statement, it is now possibe to proceed with the finite-element formulation.

The left hand side of Eq. (7) is now differentiated with respect to temperature and equated to zero to give

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \sum_{e=1}^{2m+2}\frac{\mathrm{d}I_e}{\mathrm{d}t} = 0 \quad (11)$$

It is convenient to break I_e up into its component terms.

$$\frac{\mathrm{d}I_e}{\mathrm{d}\{t\}_e} = \frac{\mathrm{d}(I_k)_e}{\mathrm{d}\{t\}_e} + \frac{\mathrm{d}(I_h)_e}{\mathrm{d}\{t\}_e} + \frac{\mathrm{d}(I_r)_e}{\mathrm{d}\{t\}_e} - \frac{\mathrm{d}(I_g)_e}{\mathrm{d}\{t\}_e} + \frac{\mathrm{d}(I_c)_e}{\mathrm{d}\{t\}_e} = 0 \quad (12)$$

Assuming a linear temperature distribution across the elements, t_e can be written in matrix from as:

$$t_e = [P]^T[P]_e^{-1}\{t\}_e \quad (13)$$

where $\{t\}_e$ and $[P]^T$ are defined in the Nomenclature.

The component terms of Eq. (12), $(I_k)_e$, $(I_h)_e$, etc., prior to differentiation can be obtained easily by comparing the corresponding terms in Eqs. (10) and (12). Differentiating each term with respect to $\{t\}_e$, with subsequent integration over the elemental length x_i to x_j, gives

$$\frac{\mathrm{d}(I_k)_e}{\mathrm{d}\{t\}_e} = \frac{1}{2}\frac{k_e b_e}{x_{ij}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\{t\}_e \quad (14)$$

$$\frac{\mathrm{d}(I_h)_e}{\mathrm{d}\{t\}_e} = \frac{h_e x_{ij}}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\{t\}_e - \frac{h_e t_a x_{ij}}{2}\begin{bmatrix}1\\ 1\end{bmatrix} \quad (15)$$

$$\frac{\mathrm{d}(I_r)_e}{\mathrm{d}\{t\}_e} = \frac{\epsilon_e\sigma(1-B_{ee})t_e^3 x_{ij}}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\{t\}_e - \frac{x_{ij}}{2}\sum_{n=1}^{2m+2}\delta_{en}\epsilon_e\sigma B_{en}t_n^4\begin{bmatrix}1\\ 1\end{bmatrix} \quad (16)$$

$$\frac{\mathrm{d}(I_g)_e}{\mathrm{d}\{t\}_e} = \frac{u_e''' b_e x_{ij}}{2}\begin{bmatrix}1\\ 1\end{bmatrix} \quad (17)$$

$$\frac{\mathrm{d}(I_c)_e}{\mathrm{d}\{t\}_e} = \frac{\rho c_p b_e}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\{\dot{t}\}_e \quad (18)$$

Note in Eqs. (14-18) that the properties have been subscripted to denote that they apply to a particular element; hence, the properties need not be constant over the length of the fin. In Eq. (16), to avoid introducing the formulation that would become a nonlinear set of equations, t_e^3 is removed from within the integration after differentiation. Since t_e is scalar and from the assumed linear temperature distribution across the elements, t_e is given as:

$$t_e = \frac{t_i + t_j}{2} \quad (19)$$

It is convenient to introduce shorthand notation for the component terms of Eq. (12) given by:

$$\frac{\mathrm{d}(I_k)_e}{\mathrm{d}\{t\}_e} = [K]_e\{t\}_e \quad (20)$$

$$\frac{\mathrm{d}(I_h)_e}{\mathrm{d}\{t\}_e} = [H]_e\{t\}_e - \{h\}_e \quad (21)$$

$$\frac{\mathrm{d}(I_r)_e}{\mathrm{d}\{t\}_e} = [R]_e\{t\}_e - \{r\}_e \quad (22)$$

$$\frac{\mathrm{d}(I_g)_e}{\mathrm{d}\{t\}_e} = \{g\}_e \quad (23)$$

$$\frac{\mathrm{d}(I_c)_e}{\mathrm{d}\{t\}_e} = [C]_e\{\dot{t}\}_e \quad (24)$$

Equations (20-24) are premultiplied by the global displacement matrix $[D]_e$, and then postmultiplied by $[D]_e^T$ to

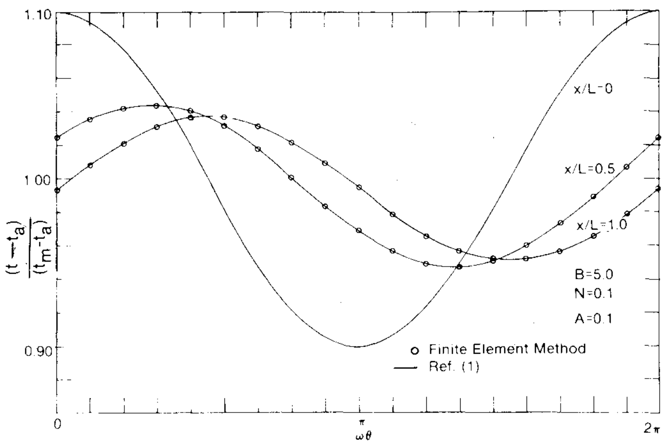

Fig. 2 Steady, periodic temperature for the limiting case of convection only.

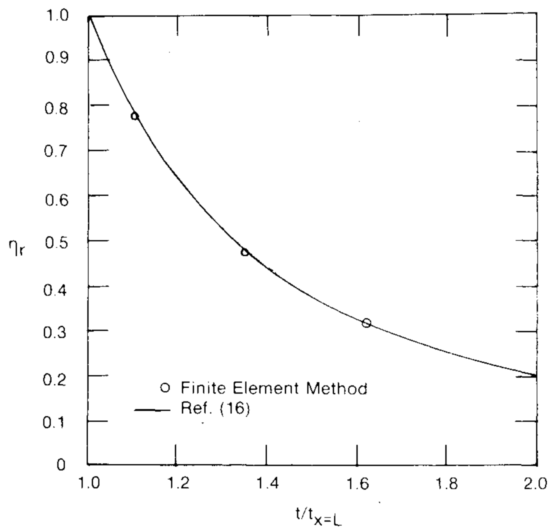

Fig. 3 Fin efficiency for the case of a single fin with a constant base temperature radiating to free space.

yield a global form. Equation (12) is now given in a compact form by:

$$[C]\{\dot{t}\} + [K+H+R]\{t\} = \{h+r+g\} \tag{25}$$

The boundary condition given in Eq. (2) is that of a specified temperature at nodal location 1, and t_1 is replaced in the global temperature vector with the expression given in Eq. (2). The boundary condition at the fin tip given by Eq. (3) is that of an insulated end. The finite-element formulation does not require any modifications to the system of equations given by Eq. (25).

Numerical Solution Technique

The Crank-Nicolson method is chosen to advance the solutions in time with iterations of the nodal temperatures $\{t\}$ occurring at each time step. The temperature-dependent elements, those containing the unknown t_e terms, were iterated in a manner such that $t_e^{(\nu+1)} \equiv (t_i^{\nu+1} + t_j^{\nu+1})/2$, until such time as the nodal temperatures converged, as prescribed by:

$$\{t\}^{(\nu+1)n} - \{t\}^{(\nu+1)n-1} \leqq \lambda \tag{26}$$

where n indicates the nth iteration at the time $\nu+1$ and λ is a specified convergence interval. For convergence within the incremental time step, λ is constant for each location along

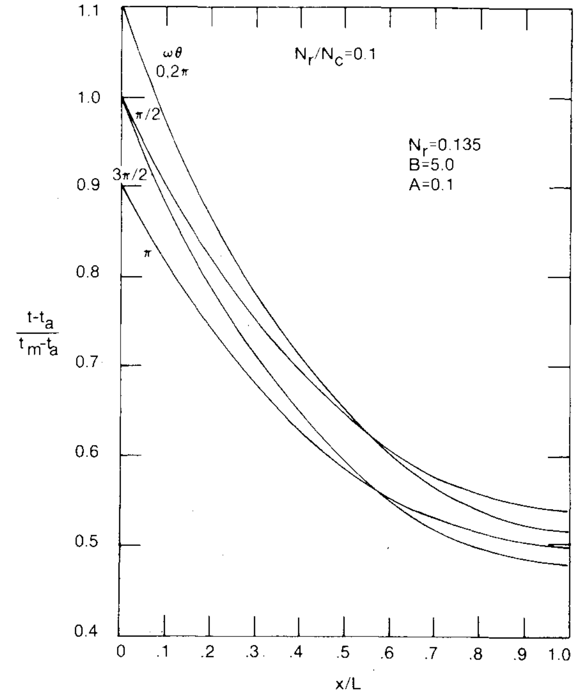

Fig. 4 Steady periodic temperature distribution in a single fin with a radiation to convection number ratio of 0.1.

the fin and is given as 0.01% of the mean base temperature. A similar convergence criterion was applied to determine at what point in the execution the temperature distributions had converged to steady periodic conditions. The solution was allowed to proceed for one complete cycle of base temperature oscillation, storing the temperature distributions. As the solution proceeded, convergence of the temperature distributions was checked, and if convergence had not occurred the temperature distribution for the previous cycle was replaced in storage with the newly calculated distribution. Once convergence was attained, the solution would proceed for one complete cycle. From the stored steady periodic temperature distributions, the instantaneous heat-transfer and fin efficiency were calculated using a five-point differentiation formula. The instantaneous efficiencies were subsequently integrated numerically over a complete cycle to give the average fin efficiency.

Results and Discussion

In order to check the accuracy of the present numerical computations, results for the temperature distributions were first compared to the limiting case of steady periodic heat transfer by convection only (i.e., $N_r = 0$) given by Yang.[1] Figure 3 illustrates the excellent accuracy obtained in calculating the steady periodic temperature distributions. It should be pointed out that the numerical computation for the instantaneous heat transfer converged in a significantly slower manner when compared to the temperature profiles. It was found that the governing numerical step size parameter p, as given by

$$p = \alpha\Delta\theta/\Delta x^2 \tag{27}$$

had to be on the order of $p = 0.02$ to obtain instantaneous heat-transfer solutions accurate to within 3% of the analytically given values of $QL/bk(t_m - t_a)$. The numerical step size parameter p will be smaller for a problem involving periodic temperature variation, as a result of the period $1/\omega$ at which the oscillations are occurring, than for a problem in which the temperature changes in a stepwise manner. $\Delta\theta$ in Eq. (27) can be rewritten in increments of the period of oscillation as shown in Eq. (28),

$$p = 2\pi\alpha / n\omega\Delta x^2 \tag{28}$$

where n is an integer representing the number of intervals that comprise one cycle of oscillation. The temperature distributions are less sensitive to p and good agreement with the analytic results is found to step size parameters as large as 1.0.

The steady-state, nonperiodic temperature distributions and resulting fin efficiencies for a single fin radiating to free space are obtained by setting N_c and A equal to zero in the computer program. Since convection is not present, the radiation fin efficiency is defined as:

$$\eta_r = Q/2\epsilon\sigma A_f t_b^4 \tag{29}$$

Heat transfer in single fin radiating to free space without surface-to-surface interaction was solved analytically by Lieblein.[16,17] The exact solution for η_r was presented implicitly. Figure 3 shows the excellent agreement between the exact solution and the present limiting solution.

The major emphasis of the study focused on the heat transfer by combined mode of radiation and convection. Two cases, in particular, were studied.

The first case was that of a single fin system with negligible base surface interaction dissipating to nonfree space. Figure 4 shows a steady-state temperature distribution for this system

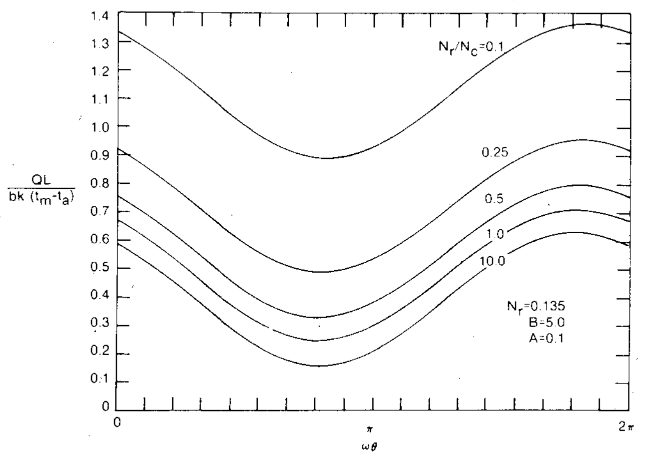

Fig. 5 Instantaneous dimensionless heat transfer for the parameter N_r/N_c for a single fin radiating to nonfree space with negligible base surface interaction.

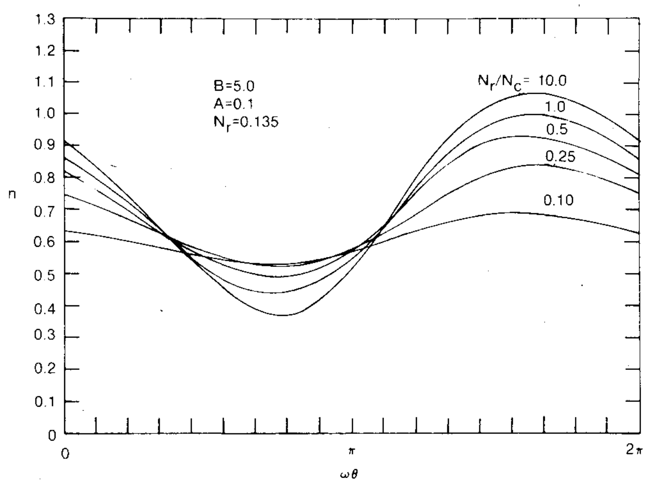

Fig. 6 Instantaneous fin efficiency for the parameter N_r/N_c for a single fin radiating to nonfree space with negligible base surface interaction.

with various values of $\omega\theta$. Figure 5 illustrates the instantaneous, dimensionless heat transfer for the parameterization N_r/N_c. Figure 6 plots the corresponding instantaneous combined mode for efficiency against $\omega\theta$. The combined mode efficiency is defined by:

$$\eta = Q/[2hA_f(t_b - t_a) + 2\sigma A_f(t_b^4 - t_a^4)] \tag{30}$$

As can be seen, the efficiency increases as radiation becomes the more dominant mechanism by which heat is dissipated from the fin, but does not reflect a net greater heat transfer from the fin. It became apparent from studying the single fin system that as the ratio of N_r/N_c becomes larger than 10, the effect on both the instantaneous heat transfer and efficiency is minimal and, as such, the system can be assumed to be purely radiative in nature. In the study, the radiation number N_r was held constant and N_c varied accordingly and, as such, the results are restricted to $N_r = 0.135$. The value of N_r chosen is representative of an anodized (high emissivity) aluminum surface at moderate temperature. A ratio of $t_m/t_a = 0.5$ is applicable for all cases presented with a mean base temperature of 1000°R. The convection coefficients were selected to be representative of the free convection range or as would exist in a rarefied atmosphere.

This combination of parameters allows for the study of systems for which a clear-cut determination of the major mechanism by which heat is dissipated from the system is not readily apparent. The parameterization of N_r/N_c is chosen since it represents what can be viewed as a ratio of a radiative heat-transfer coefficient to a convective heat-transfer coefficient. The similarity to a heat-transfer coefficient is noted by Sparrow et al.[5]

The second case studied involved heat transfer from a fin array for which it was desired to determine the effect of the fin pitch spacing given as d/L.

Considering the combined mode heat transfer from an array of fins forming a gray, nonisothermal enclosure, it is necessary to determine the elemental absorption factors first. The absorption factors for the $m = 20$ finite-element system representation shown in Fig. 1 are illustrated in Figs. 7 and 8. Figure 7 shows the fraction of emission from a given element located on the same fin that is absorbed by a given element e and is a graphical representation of $dB(x,x)$. Figure 8 is similar to Fig. 7, but gives the absorption factor from fin elements located on the two adjacent opposing fins. It is evident that the absorption factors from an element to itself and from a directly opposed element on an opposing fin are always maximum. This is a result of the reflected emission by

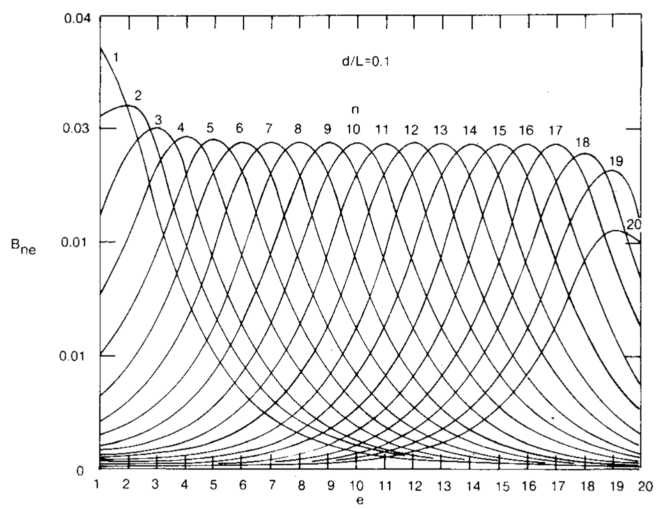

Fig. 7 Absorption factors for a given element e to elements located on the same fin.

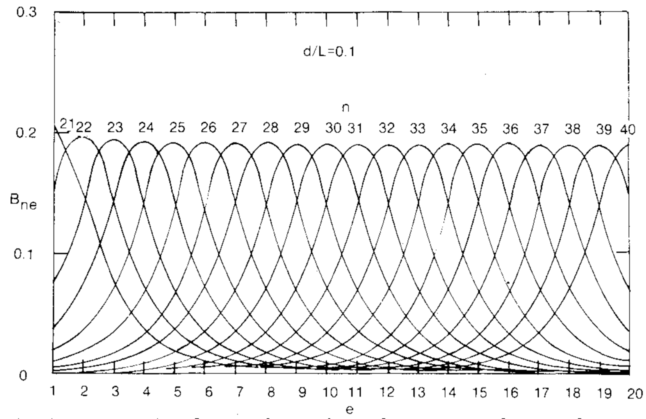

Fig. 8 Absorption factors for a given element *e* to elements located on the opposing fin.

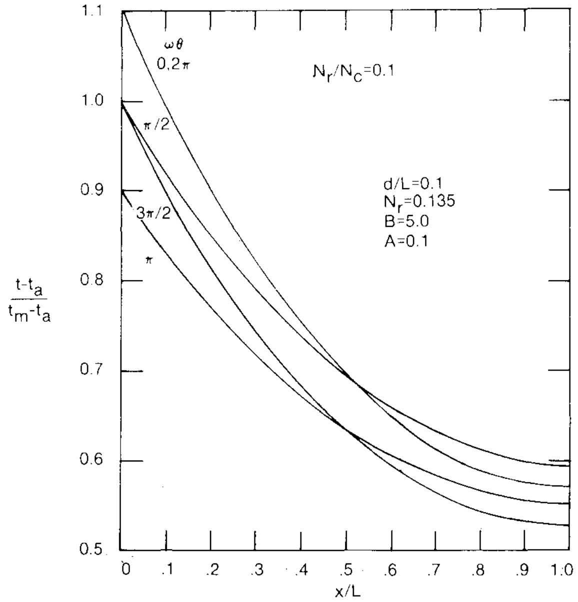

Fig. 9 Temperature distribution for a fin array of pitch spacing $d/L=0.1$.

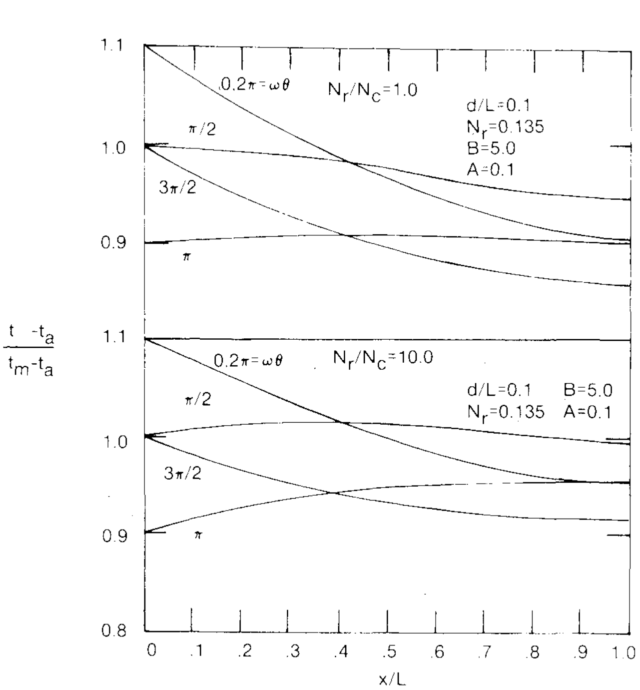

Fig. 10 Temperature distribution for a fin array of pitch spacing $d/L=0.1$.

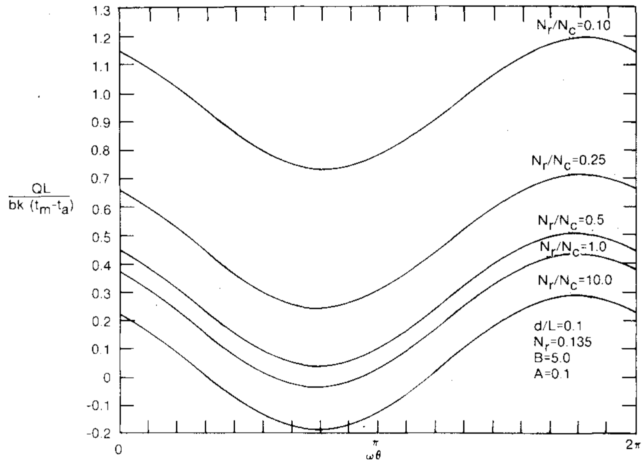

Fig. 11 Dimensionless instantaneous heat transfer from a fin array for $d/L=0.1$.

the *e*th element always being maximum from the directly opposed element, by virtue of radiation shape factor to the opposed element being maximum. Elements 1, 2, and 3 exhibit larger absorption factors due to reflection by the base surface, which is assumed to have the same emissivity as the fin. As an additional check on the numerically calculated values, it was found that the absoption factors for all elements forming the enclosure sum to unity, i.e.,

$$\sum_{n=1}^{2m+2} B_{en} = 1$$

The absorption factors shown in Figs. 7 and 8 are for a fin pitch spacing of $d/L=0.1$. For pitch spacings other than 0.1, it is observed that the absorption factors exhibit the same characteristics. Figures 9 and 10 illustrate the steady, periodic temperature distributions for $d/L=0.1$. The temperature distributions, when a pitch spacing of $d/L=1.0$ is considered, exhibit the same characteristic shape but are of lower amplitude due to the increased dissipation of heat by radiation to the environment. Not until the ratio of N_r/N_c becomes ≥ 0.25 is it found that the temperature distribution for either pitch spacing is significantly affected by radiation. Comparison of Figs. 9 and 4 illustrate that the temperature distributions have increased significantly over the distributions for the single fin. Figures 11 and 12 compare the net instantaneous dimensionless heat transfer for the cases of $d/L=0.1$ and 1.0, respectively. The fin system with pitch spacing $d/L=1.0$ gives consistently higher values of the instantaneous heat transfer and efficiency than for the system with $d/L=0.1$. It is interesting to note that negative values of instantaneous heat transfer and fin efficiency exist. The negative values indicate a backflow of heat from the fin into the base surface and is consistent with the similar behavior noted by Yang[1] in the study of purely convective fins.

A final comparison of the systems studied is presented in Fig. 13. Both steady nonperiodic and steady periodic fin efficiency over one cycle of base temperature oscillation is shown. The steady nonperiodic efficiencies are based on the mean temperature of the base surface. As illustrated in Fig. 13 for the nondimensional frequency parameter $B=5.0$, the steady periodic fin efficiencies are closely approximated by the steady-state values at the mean base temperature. The most significant aspect of Fig. 3 is the degree to which the steady-state fin efficiencies closely approximate the steady periodic efficiencies. However, the present computations revealed that the computer time required to obtain steady, periodic efficiencies as opposed to steady nonperiodic occurs in a ratio of approximately 90:1.

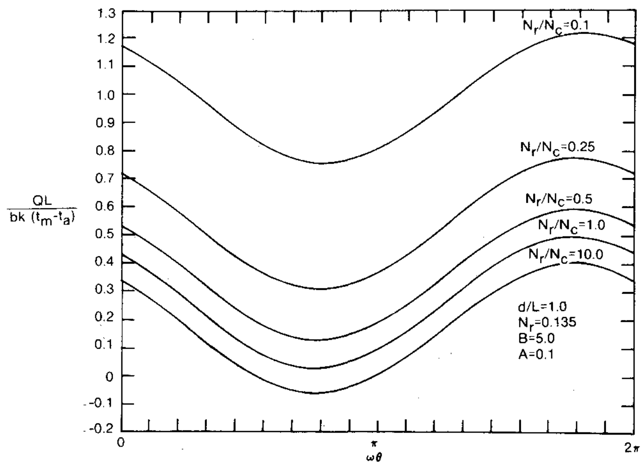

Fig. 12 Dimensionless instantaneous heat transfer from a fin array for $d/L = 1.0$.

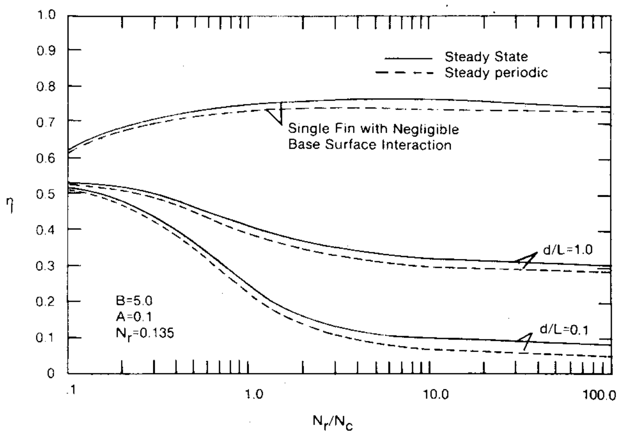

Fig. 13 Steady-state and steady periodic combined mode average fin efficiencies vs the parameter N_r/N_c.

Conclusion

A finite-element formulation is developed to permit the study of combined mode heat transfer from a finned system with a periodic base temperature. Comparisons to analytically given limiting cases illustrate that the present numerical solutions agree well with analytic results. Emphasis has been placed on the study of heat transfer occurring in a steady, periodic manner with several cases having been presented. While this work has been restricted to considering primarily the fin efficiency for constant properties, the computer program can be easily modified to incorporate variable properties and other more complicated fin geometries. It should be pointed out that the same problem can be solved using finite-difference methods. The major advantages of finite-element techniques are that: 1) the irregularly shaped region can be handled easily without special treatment and 2) the size of finite element can be varied over the region, permitting the use of small element without bookkeeping difficulty, as is usually encountered in finite-difference methods. Therefore, the finite-element technique is employed in the present study with the intention that the computer program can be easily extended to handle the more complicated fin geometries.

Due to a large number of parameters involved in the study, the presentation of an extensive parametric study becomes prohibitive, as the space is limited. In addition, the computer run times for each combined mode case are, in general, quite lengthy. However, the computer programs are well documented in the Appendix of Ref. 18.

Acknowledgment

The fellowship granted by Bendix Corporation to R. Eslinger is gratefully appreciated.

References

[1] Yang, J. W., "Periodic Heat Transfer in Straight Fins," *Journal of Heat Transfer, Transactions of the ASME*, Vol. 94, Aug. 1972, pp. 310-314.

[2] Aziz, A., "Periodic Heat Transfer in Annular Fins," *Journal of Heat Transfer, Transactions of the ASME*, Vol. 97, May 1975, pp. 302-304.

[3] Eckert, E.R.G. and Drake, R. M., *Heat and Mass Transfer*, McGraw Hill Book Co., Inc., New York, 1959.

[4] Schnurr, N. M. and Cothran, C. A., "Radiation from an Array of Gray Circular Fins of Trapazoidal Profile," *AIAA Journal*, Vol. 12, Nov. 1974, pp. 1476-1480.

[5] Sparrow, E. M., Eckert, E.R.G., Ervine Jr., T. F., "The Effectiveness of Radiating Fins with Mutual Irradiations," *Journal of Aerospace Sciences*, Vol. 28, Oct. 1961, pp. 763-778.

[6] Sparrow, E. M., Miller, G. B., and Jonsson, V. K., "Radiating Effectiveness of Annular-Finned Space Radiators, Including Mutual Irradiation Between Radiator Elements," *Journal of Aerospace Sciences*, Vol. 29, Nov. 1962, pp. 1291-1299.

[7] Sparrow, E. M. and Eckert, E.R.G., "Radiant Interaction Between Fin and Base Surfaces," *Journal of Heat Transfer, Transactions of the ASME*, Vol. 84, Feb. 1962, pp. 12-18.

[8] Wilson, E. L. and Nickell, R. E., "Applications of the Finite Element Method to Heat Conduction Analysis," *Nuclear Engineering and Design*, Vol. 4, May 1966, pp. 276-286.

[9] Myers, G. E., *Analytical Methods in Conduction Heat Transfer*, McGraw-Hill Book Co., Inc., New York, Aug. 1971.

[10] Padovan, J., "Steady Conduction of Heat in Linear and Nonlinear Fully Anisotropic Media by Finite Elements," *Journal of Heat Transfer, Transactions of the ASME*, Vol. 96, Aug. 1974, pp. 313-318.

[11] Emery, A. R., "The Use Singularity Programming in Finite Difference and Finite Element Computations of Temperature," *Journal of Heat Transfer, Transactions of the ASME*, Vol. 95, Aug. 1973, pp. 344-351.

[12] Chu, S. C. and Yalamanchili, R.V.S., *Application of the Finite-Element Methods to Heat-Transfer Problems*, Parts I and II, Tech. Rept. AD 726370 and AD 726371, U.S. Army Weapons Command, 1971.

[13] Richard, P. D. and Shum, Y. M., "Use of Finite Element Methods in Solution of Transient Heat Conduction Problems," ASME Paper 69-WA/HT-36, 1969.

[14] Beckett, R. E. and Chu, S. C., "Finite Element Method Applied to Heat Conduction in Solids with Nonlinear Boundary Conditions," *Journal of Heat Transfer, Transactions of the ASME*, Vol. 95, Feb. 1973, pp. 126-128.

[15] Gebhart, B., "Unified Treatment for Thermal Radiation Transfer Processes—Gray, Diffuse Radiator and Absorbers," ASME Paper 57-A-34, 1957.

[16] Lieblein, S., "Analysis of Temperature Distribution and Radiant Transfer Along Rectangular Fin of Constant Thickness," NASA TN D 196, 1959.

[17] Kern, D. O. and Kraus, A. D., *The Extended Surface Heat Transfer*, McGraw-Hill Book Co., New York, 1972.

[18] Eslinger, R. G., "Finite Element Analysis of Periodic Heat Transfer in Radiating and Convecting Fins," M.S. Thesis, Department of Mechanical Engineering, University of Akron, Akron, Ohio, 1977.